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Assets and liabilities are the momentum of particles and antiparticles displayed in Feynman-graphs

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Abstract

An analogy between assets and liabilities and the momentum of particles and antiparticles (called actons and passons) is proposed. It allows physicists to use physical methods in economy for the analysis of monetary systems and for the analysis of double entry bookkeeping. Economists can use it to subdivide and discuss complicated balance transactions in terms of Feynman-graphs which introduce the time dimension to bookkeeping. Within the analogy, assets and liabilities come into existence by pair creation. Conservation of momentum is fulfilled whereas the conservation of energy corresponds to the regulation of a constant amount of money. Interest rates accelerate the particles by imposing a negative friction. The statistical description of an ideal money gas is derived and the transcription to semiconductor physics is given. The analogy is hoped to open a new field for physics and to reveal new insights on monetary systems.

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1. Introduction

Crises of monetary systems have plagued mankind not only in the last century between the two world wars, but still in our modern times. Monetary systems suffer from a cursorily understanding of the underlaying mechanisms. The description of money and monetary systems is still qualitative. Quantitative equations and definitions are not precise and have failed to describe money. For example, the definition of the amount

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of money by the german central bank began with money stock M1, has included then M2, was extended by M1B and M3 and lately by M3erweitert [1,2]. The swiss central bank has ultimately given up to define it [3].

The author presents a novel bridge from physics to economy in form of an analogy between assets and liabilities and the positive and negative momentum of particles. It opens the way for statistical mechanics as applied to money and allows the description of complicated money or balance transactions in Feynman-graphs in space–time. Equations for an ideal money gas are discussed. The analogy stresses the symmetry between assets liabilities. The approach of the analogy is expected to help in clarifying the basis of money and debt by discussing it from a new viewpoint.

2. Analogy between money and momentum

2.1. Basics of the analogy

The momentum of particles along one direction is assigned to assets or liabilities, depending on the sign of the momentum. Assets (‘money’) are assigned to particles with positive momentum called actons, liabilities (‘debt’) to particles with negative momentum called passons. Their movement are displayed in Feynman-graphs (Fig. 1). The conservation of momentum is fulfilled if money or debt is not faked.

2.2. Ownership and transfer

To distinguish ownership, a second dimension is attributed to the particles, indicated by the color of the arrow head in the Feynman-graphs. Particles cannot change their ownership. For example the transfer of money from A to B is implemented by striking a resting acton of B by a moving acton of A (Fig. 2a). The transfer of debt can be accomplished the same way with passons with inverted space axis of the graph (not shown). Resting particles which indicate no value are often omitted in the Feynman-graphs as they do not interact and represent no value. The corresponding footprint in a double entry bookkeeping (assets on the left, liabilities on the right) is shown in Fig. 2b. Also shown is the running total in a third column as this corresponds

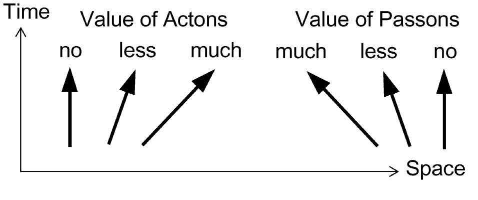

Fig. 1. Money or assets of different value are acton particles with positive momentum shown in a Feynman-graph of space–time. Debts or liabilities are passon particles with negative momentum.

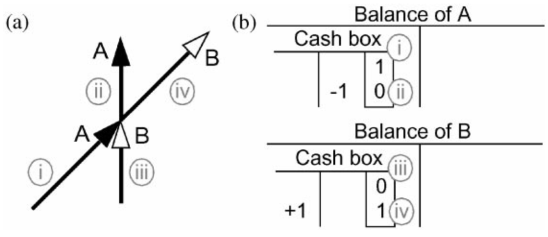

Fig. 2. Transfer of money from A to B. (a) Feynman-graph of the process: A's moving acton/passon strikes B's resting particle. The arrow head marks the owner. Inverting the space axis gives debt transfer between passons from A to B. (b) Double entry bookkeeping of the process. Correspondence between the running total of the balance and the arrows are given by numbers in circles.

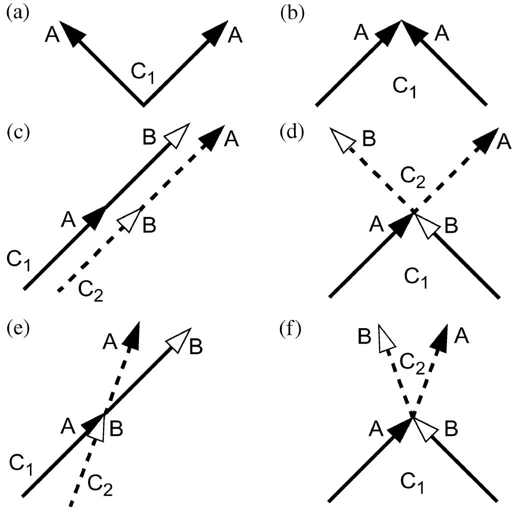

Fig. 3. Pair creation and currencies. Pair creation (a) and annihilation (b) of the currency C_1. (c,d) Exchanging between C_1 and C_2 by A and B in two ways. (e,f) Same with an exchange course $C_1 : C_2 = 2 : 1$.

directly to arrows and the momentum in the graph which is indicated by numbers in circles.

2.3. Pair creation, currency and exchange

Acton/passon pairs can be created and annihilated at any time and not only for small times as governed by the uncertainty relation of energy and time of virtual particles (Fig. 3a and b). Energy is not conserved. Energy is two times the amount of money where the latter is defined by the amount of bank liabilities. Therefore, the conservation of energy corresponds to the regulation of the amount of money. To stress the origin of a money/debt pair, we assign each pair creation a new currency, marked by the color of the arrow haft. The currency is strictly speaking not necessary as one can

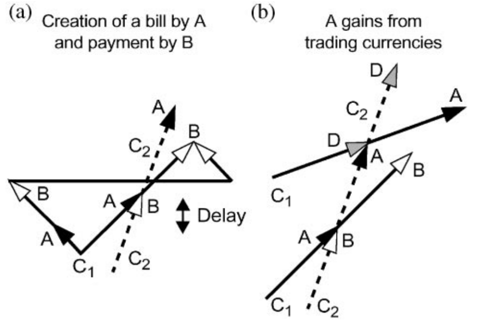

Fig. 4. Examples. (a) Creation and delayed payment of a bill. (b) Gaining from trading currencies.

in principle always track the origin of debt or money. But this gives a more precise definition of a currency. Note that even within a country, we can find many different currencies as acton/passon pairs can be created by banks or stock companies. Therefore, a currency marks the relationship between actons and the corresponding passons. This must not be confused with the ownership of the passon which indicates the person which is responsible for covering the corresponding acton defined by the currency. (However currencies are often grouped by contracts, for example the accounts of a bank are grouped to the currency of the bank.) So far mentioned all processes are only allowed for particles of the same currency. By definition, pair creation and annihilation is only allowed for actons/passons with the same momentum, owner and currency. But the currencies can be exchanged. Two actons (or two passons) of different ownership and currency (Fig. 3c) or one acton and one passon of different ownership but same currency can be exchanged (Fig. 3d). Both processes allow different momenta between the currencies which means that the angle between arrows in the Feynman-graph indicate the exchange course of the currencies at that time (Fig. 3e and f). Here, the currency C_2 is twofold more valuable than C_1 at that time for the persons A and B. We see that momentum is conserved not only overall, but also within a currency.

2.4. Examples

We want to demonstrate the notions of ownership and currency exchange by two examples. The first is a delayed payment of a bill (Fig. 4a). Assume A has done something for B, therefore A is writing a bill. This creates a new currency C_1 with a passon (the bill) and an acton (let us say the carbon copy of the bill). A gives the passon to B and thus pays by transfer of liabilities. B has now the duty to cover the bill-currency C_1. B is 'paying' the bill by actually exchanging C_1 with another currency C_2 after some delay, e.g. 30 days in Germany. Now B has both acton and passon of the bill and is able to annihilate the currency again.

Currencies can be traded. A is owning currency C_2, can trade it cheap with B into C_1 and afterwards cheap with C back into C_2 (Fig. 4b). Either the different subjective's

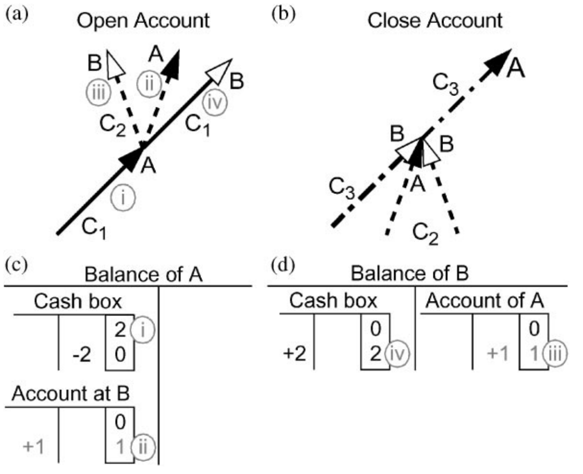

Fig. 5. Bank accounts. (a) Opening an account by A at Bank B. (b) Closing the account. (c) Double entry bookkeeping of the account opening if we allow the different currencies C_1 and C_2 in the same balance indicated by grey numbers for C_2. (d) Double entry bookkeeping of the account closing.

judgement of the currencies (expressing the reliability of the owners of the passons of C_1 and C_2) or the time between the two changes has made different exchange courses possible (e.g. by inflation).

2.5. Bank accounts

The opening of a bank account creates bank money. It is a combined process of pair creation of bank money and an exchange of the currencies. Bank B creates an acton/passon pair in currency C_2 and exchanges it with the acton in currency C_1 which A is providing (Fig. 5a). A will from now on rely on the passon held by the bank in C_2 and the bank on the passon of C_1 (which is not drawn in the graph as it originates from an earlier pair creation process). In Fig. 5c the process is translated to double-entry bookkeeping, again with a running total in a third row which corresponds to the particles by numbers in circles. Note that we write two currencies in the same balance, indicated by different grey scales, which is not common. The time and space reversed process of the opening depicts the closing of the account where bank money is annihilated (Fig. 5b). There is no guarantee that A gets back his account in the original currency C_1, in the example he will obtain C_3. Also the exchange course is not guaranteed. If A obtains a credit from the bank B, the process is the same, but the subjects are exchanged. Now A is 'the bank' which creates a new currency with an account and B gives the currency which is exchanged by A. All account processes have only be shown with initial actons, the same processes could also be initiated by passons in spacially inverted graphs stressing the symmetry between actons and passons.

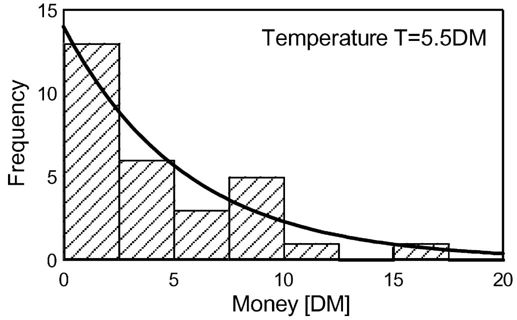

Fig. 6. Does cash or actons follow a Boltzmann distribution? Given is the histogram of the expenses of the author while 14 days. It can be fitted by a Boltzmann distribution with a temperature of 5.5 DM. But this only shows a small subgroup of all actons in the monetary system.

3. Statistical money mechanics

Using the laws and state variables of statistical mechanics, we can deduce physical laws describing monetary systems. It is difficult to obtain statistical data of monetary systems in space–time at the level of detail needed. Typically, not too much is known about the ownership and transfer of money if we do distinguish money origins (currencies) as done here. This loss of information of the origin due to mixing of the currencies is an important, but in extreme situations also dangerous insurance effect within monetary systems. The laws presented here are not yet applied to real monetary systems and effort should be undertaken to test the following statistical mechanics of particles.

3.1. Energy distribution

From distributing the energy into canonical microstates, we find the Boltzmann distribution of energy

$$n(E) = \exp(-E/kT) \quad (1)$$

with temperature T and the unit converting Boltzmann factor k which can be set to 1 if T is measured in energy. It is well possible that monetary systems demand for other distributions the same way quantum mechanics needs Fermi- or Bose-distributions. A test of (1) can be found in Fig. 6, where the expenses of the author for 14 days can be readily fitted by a Boltzmann distribution with a temperature of $T = 5.5$ DM. Note that this is not a representative subgroup of all cash flows as for example the income has a much higher temperature. Also, it is not clear whether the used time average gives the same results as the ensemble average. It is not easy to obtain cash and balance data of a representative subgroup of a whole monetary system.

3.2. Ideal money gas

We will use the linear relationship $E=|p|$ between momentum and energy to linearly assign the amount of money to half of the total energy of the system. We obtain the same units for energy and momentum by using natural units known from particle physics. We will use Planck's constant $h = 1$. The known equations for the ideal gas in one dimension have to be reconsidered on base of the linear energy-momentum relationship. The partition function Z of a single particle now yields the expected value of the momentum:

$$Z \propto \int_{-\infty}^{\infty} \exp(-|p|/kT)\,\mathrm{d}p, \quad \langle E\rangle = \langle |p|\rangle = 2kT\,. \tag{2}$$

Therefore, the specific heat capacity per particle is constant with $C_V=2k$. The volume of the gas stretches from 0 to L. To obtain a density of states $D(E)$, we just take over quantum mechanical arguments. Using the maximum momentum $p_{\max} = n_{\max}/2L$ with the maximum quantum number $n_{\max}$ we obtain

$$D(E) = \mathrm{d}N/\mathrm{d}E = 2L\,. \tag{3}$$

Note that bringing the length into the analogy is the implicit assumption that the probability P of striking another particle depends linearly on the momentum p (which has to be tested for money):

$$P = \sqrt{2}cqp/m, \quad \Lambda = (\sqrt{2}cq)^{-1}\,, \tag{4}$$

where c is the density of particles per length, q is the scattering cross section and m the mass of a particle [4]. Λ depicts the mean-free path which is not expected to depend on the momentum. If (4) does not hold for money, we will have to reinterpret the notion of length in the analogy. Based on these remarks concerning the length are the conduction and diffusion of heat and the state equation. With the diffusion coefficient D we can define the conduction of heat j:

$$D = \frac{\langle |p|\rangle}{m}\Lambda, \quad j = cDC_V\nabla T\,. \tag{5}$$

Evaluating the momentum transfer at the wall of the gas as e.g. done in Ref. [4], we find the pressure p_r giving the state equation of money gases:

$$p_r = \frac{c}{m}\langle p^2\rangle = 2\frac{c}{m}(kT)^2\,. \tag{6}$$

3.3. Semiconductors: band gap and defects

The analogy of actons and passons can be transformed to electrons (n-particles) and electron holes (p-particles) in semiconductor physics. Many scenarios of pair creation and annihilation can be found in the literature. For example, if we create pairs over the band gap thermally, the amount of pairs at temperature T is controlled by the band gap E_G. With the constant density of states $D(E)$ from (3) and the Boltzmann statistic, shifted by the band gap, we obtain the particle densities per length of actons A and of passons P:

$$A = 2kT \exp\left(-\frac{(E_G - E_F)}{kT}\right) ,$$

$$P = 2kT \exp\left(\frac{-E_F}{kT}\right) \tag{7}$$

and therefore the intrinsic concentration c per length

$$c = \sqrt{AP} = 2kT \exp\left(\frac{-E_G}{2kT}\right) . \tag{8}$$

If we do not have defects, we can infer from $A=P$ the Fermi-energy level $E_F=E_G/2$. A defect means that actons or passons do not interact with other particles any longer, meaning that in the case of trapped actons, the money is hoarded. If passons are trapped in the analogy, debts cannot be cleared.

3.4. Virtual particles and renormalization

The uncertainty relation between energy and time allows the creation of virtual particle/antiparticle pairs for small times. The subsequent annihilation is favoured by interest rates in a money system, but typically not forced after specific time spans as in physics. The uncertainty relation allows for different energy-time pairs: much energy for small times and less energy for long times. Note that the number of virtual pairs which are created is not defined by first principles in quantum mechanics and relates to the renormalization problem [5–7].

4. Economist's viewpoints

Amount of money: One of the main problems of monetary systems is the regulation of the amount of money. We see in the analogy that the total energy of the money particles of a currency is the twofold amount of money in a system. We see that the creation and annihilation of money is possible without further boundary conditions as the total energy is not conserved. Mechanisms have to be introduced to conserve and regulate the total energy.

Inflation: An increased money amount does increase the price level. We can see an increased money amount by comparing with a second 'standard' currency by informed and competing subjects. The space–time of Feynman-graphs shows that we can only compare values at one time.

Currencies: Within our general notion of currencies, every object is a currency as every object has some value. Of course not all objects are equally exchangeable which means a loss of value due to the reduced liquidity. Also the exchange course must account for change in value of an object as some objects degrade in time, some get more valuable and some increase value if subjects work with them.

Pair creators: Not only commercial or central banks are allowed to create currencies by pair creation. With our general notion of a currency, a bill is also a currency, although it is typically not as exchangeable or transferable as other currencies. The basic pair creator is nature – and mankind should include itself. This should be kept in mind although it might be a tautology.

Interest rates and dissipation: An exponential money growth from successive application of interest rates corresponds to a negative dissipation or friction in a medium. Forcing an overall negative interest rate could be a way to regulate the total energy.

Double entry bookkeeping: The transcription of Feynman-graphs of this analogy to the double-entry bookkeeping will be further examined in a subsequent paper [8]. The one to one correspondence is already indicated in Figs. 2 and 5. Therefore, the Feynman-graphs are directly transferable into double entry bookkeeping and vice versa. The notion of different currencies introduced here are implemented into double entry bookkeeping in Fig. 5c although different currencies are not mixed usually in balances.

5. Conclusions

We have discussed a novel analogy between money and momentum in the light of Feynman-graphs, double entry bookkeeping, conservation of energy and momentum, the uncertainty relation, statistical mechanics, semiconductor physics and economics. The concept of momentum discussed as money or debt can hopefully help to find new crucial state variables of monetary systems. Monetary systems are too essential and too important not to be understood quantitatively in more detail.

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References

[1] O. Issing, Einführung in die Geldtheorie, 11th Edition, Verlag Franz Vahlen, Munich, Germany, 1997, ISBN 3-8006-2328-5.

[2] H.J. Jarchow, Theorie und Politik des Geldes, 10th Edition, UTB für Wissenschaft, Stuttgart, Germany, 1998, ISBN 3-8252-0234-8.

[3] M. Boemle, P. Ehrsam, M. Gsell, P. Nyffeler, E. Rutschi, Handbuch des Geld-, Bank- und Börsenwesens der Schweiz, Fourth edition, Ott Verlag, Thun, Switzerland, 1996, ISBN 3-7225-6328-3.
[4] W. Göpel, H.D. Wiemhöfer, Statistische Thermodynamik, Spektrum Akademischer Verlag, Heidelberg, Germany, 2000, ISBN 3-86025-278-X.
[5] S. Weinberg, The Quantum Theory of Fields, Vol. 1, Cambridge University Press, Cambridge (Chapter 13).
[6] F. Bloch, A. Nordsieck, Phys. Rev. 37 (1937) 54.
[7] D.R. Yennie, S.C. Frautschi, H. Suura, Ann. Phys. NY 13 (1961) 379.
[8] D. Braun, R. Fischer, Money as momentum: Bookkeeping of a bank transcripted to Feynman-graphs, to be submitted.