Bookkeeping Mechanics

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Double entry bookkeeping is translated to the momentum exchange of bouncing particles given in space-time graphs. This translation defines momentum, force and energy of bookkeeping. Bookkeeping-momentum is conserved whereas bookkeeping-energy is not. Currencies originate from particle pair creations. Liability-particles are equivalent to asset-particles moving backwards in time. Bookkeeping is axiomatically deconstructed into basic transfer and exchange graphs. In banking, we find a hidden exchange rate between an account currency and a loan currency. We find zero-sum game violations in statements of capital, depreciations of tangible assets and single-sided exchanges.

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Bookkeeping is the mandatory measurement method for money and therefore the basis of modern economics. We analyze bookkeeping with a tight physical analogy which directly connects the main field of theoretical physics which is the interaction of particles, to the backbone of economics, namely double entry bookkeeping. This yields a description of bookkeeping in space-time graphs of bouncing particles. We call this physical inspired analysis bookkeeping mechanics. It allows for an easy and precise discussions of complicated intertemporal and multicurrency bookkeeping transactions. Bookkeeping mechanics does not presume economic modelling of prices or exchange rates. It merely provides a precise framework to develop quantitative economic models on the basis of double entry bookkeeping.

We divide the analysis into four parts. First, we derive and discuss the translation from assets and liabilities of bookkeeping to positive and negative momentum of bouncing particles in one dimension. Second, bookkeeping is deconstructed into a systematics of exchange and transfer possibilities between up to four subjects using one, two or three currencies are derived. This forms the basis to analyze in the third part the bookkeeping mechanics of basic banking transactions. In the fourth part, we show how the non-monetary bookkeeping of statements of capital, of depreciations and of single-sided exchanges surprisingly fail to obey the zero-sum game.

1. Translation

It is commonly accepted that a major driving force in the natural sciences are analogies (Axel Tiemann (1993)). Historical scientific texts are often filled with comparisons which are used to approach a novel phenomena. Since the early days of economics, analogies were discussed to model the behavior of markets, a prominent example is the price-force balance of Irving Fisher's Ph.D. thesis (Irving Fisher (1891)). Yet we will approach bookkeeping here at the fundamental physical level of momentum and will exclude direct economic modelling.

Double entry bookkeeping (Luca Pacioli (1494), Arthur Schultz (1943), Jürg Leimgruber (1992)) is the statutory measurement method of money (Emilio Albisetti et.al. (1996), Otmar Issing (1997), Hans Schmid (1997), Georg Obst and Otto Hintner (2000), Jürgen Krumnow and Ludwig Gramlich (2000)). In this sense, our physical analysis of bookkeeping will be a re-interpretation of money from the point of view of theoretical mechanics. By translating between the two logical worlds, we will find new approaches to the foundations of monetary systems and the logical basis of transfers and exchanges. It makes economics accessible from the physics of statistical mechanics at the very basic level. Bookkeeping mechanics does not model prices or exchange rates, but constructs the qualitative scaffold behind both. Since the mechanical picture includes time as an integral part, intertemporal analysis can be performed easily.

Although the basis of the translation was already described (Dieter Braun (2001)), we will extend and restate its foundation here. In a nutshell, classical bookkeeping is an adding and subtracting scheme of assets and its negative counterpart liabilities in a common currency. We will extend bookkeeping to track many currencies. We find that an insightful adding scheme to compare bookkeeping with is the momentum of particles. As the particles bounce, the conservation of momentum ensures that any subtracted or given momentum will be added or received by another particle. We will therefore associate positive momentum units of particles to the right in one dimension with asset currency units and associate negative momentum units to the left with liability currency units. For simplicity, we assume that all particles have the same mass, which means that only the velocity of a particle determines its momentum.

It is common in physics to discuss particle paths in space-time graphs (Hermann Minkowski (1952)). They are called Feynman-graphs in quantum electrodynamics (Richard Feynman (1985)). In space-time graphs, one dimension of space is plotted to the right and the time axis is directed upwards

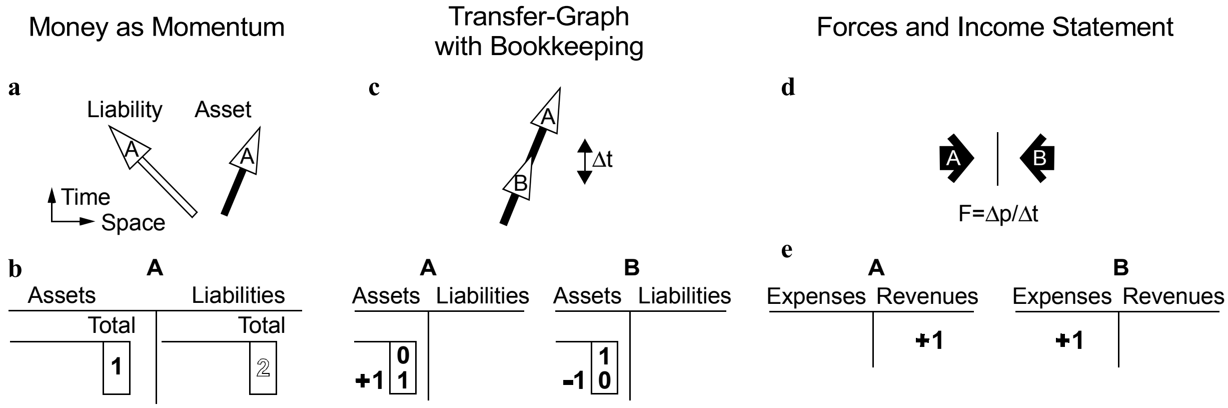

Fig. 1. ***Translation of bookkeeping mechanics***. (a,b) An asset (liability) is a particle with positive (negative) momentum p to the right (left) displayed in a space-time graph. We show an asset of one black currency unit and a liability of two white currency units together with the bookkeeping. (c) Asset transfer graph. B is transferring assets to A, shown in the space-time graph and the bookkeeping. (d,e) Income statement. The momentum change Δp within time Δt for each subject and currency yields a force balance F=Δp/Δt. It can be deduced from the graph directly and refers to expenses (to the left) and revenues (to the right) of the income statement.

(Fig. 1a). We obtain a complete recording of particle movements: as time goes up, resting particles yield a vertical trace which tilts to the right for particles moving to the right. The more horizontal the path is, the more momentum the particle has, since more space units are crossed in the same time. Particles moving to the right symbolize assets, noted on the left side of bookkeeping. Paths to the left are liabilities, noted on the right side of the bookkeeping. An asset of one black currency unit owned by subject A and a liability of two white currency units owned by A are given in the space-time graph (Fig. 1a) and the bookkeeping (Fig. 1b). The asset and liability currency units are noted in a box of running total, left from it we note the change of asset or liability. Two more qualities are associated with a particle. First the subject owning the particle, indexed by i, which is also the owner of the bookkeeping. Owners are marked by letters in the arrow heads in the graph. Second the currency, indexed by c and marked by the color scheme of both the particle path (arrow haft) and the coloring of the numbers in the bookkeeping. As shown, we extend classical bookkeeping by allowing different currencies in the same bookkeeping.

Consider B giving one black asset currency unit to A. In bookkeeping mechanics, this corresponds to A's particle bouncing at a specific time a resting particle of B, shown in the space-time graph and the bookkeeping in Fig. 1c. The particle of B is moving to the right and hitting the particle of A at a specific time, i.e. height in the graph. Note that for clarity, resting particles which have no value are not shown in the graph, therefore the bouncing partner is not visible in the graph before the collision. Also, particle paths could be moved horizontally in zero time to adapt the path for collision partners, since its slope and therefore its momentum is not changed by this. In the bookkeeping, the time increases downward from row to row. A revenue (+1) for A increases the asset whereas a expense (-1) for B decreases it. The boxed balance field shows the running total and is placed right from the expense/revenue changes. We see the addition of an asset to A's bookkeeping while it is subtracted from B's. We summarize:

> ***Translation Axiom of bookkeeping mechanics.*** Asset is positive particle momentum to the right of equally massed particles along one dimension. Liability is negative particle momentum to the left. Particles are further characterized by ownership i and currency c. The momentum p_{ic} is given in currency units . We display the particle momentum by trajectory arrows in space-time graphs. Ownership i is marked in the arrowhead and currency c by the color of the arrow haft. Resting particles have no

value and are not displayed. We call asset particles actons and liability particles passons. All bookkeeping information is thus contained in the graph.

We can draw several physically motivated conclusions from this translation axiom.

Conclusion 1: Momentum conservation. If bookkeeping across several owners follows the rules of a zero-sum game, that is, each asset or liability given in a currency is also received, we find that the sum over all momentums of subjects i is constant for each currency c:

$$\sum_i p_{ic} = \text{const.} \qquad \forall c \tag{1}$$

Conclusion 2: The income statement is a force balance. Double entry bookkeeping restates changes of assets and liabilities as expenses and revenues (Fig. 1e). The momentum change $\Delta p_{ic}=p_{ic}(t+\Delta t)-p_{ic}(t)$ is an expense for $\Delta p_{ic}<0$ and a revenue for $\Delta p_{ic}>0$. An income statement thus adds expenses and revenues of a time span Δt. It appears useful to divide by Δt to obtain an average change of asset and liability per time unit. By Newton's second law of mechanics, this is the definition of a force: $F_{ic}=\Delta p_{ic}/\Delta t$. Thus an income statement is a force balance between subjects within the time span Δt. Expense forces point to the left and slow down particles with positive momentum (assets) or accelerate particles with negative momentum (liabilities). Revenue forces on the other hand point to the right and slow down particles with negative momentum (liabilities) or accelerate particles with positive momentum (assets). The force balance for the transfer is shown in Fig. 1d. We see that the forces of the income statement are derived from the particle trajectories and yield no additional information.

Conclusion 3: Zero force sum. We can restate the zero-sum game of (eq. 1) by time derivation as a zero net force balance of the income statement:

$$\sum_i F_{ic} = 0 \qquad \forall c, \Delta t \tag{2}$$

Thus the sum over the income statement forces of all subjects i is zero for each currency c and all possible time intervals Δt.

Conclusion 4. Creation of currencies. Currencies are the units of assets and liabilities which can add and subtract directly without exchanging. To obtain assets and liabilities consistent to the previous rules, they have to be created in pairs of positive and negative momentum with equal number of currency units and equal ownership. In order not to vanish into pair annihilation immediately after their creation, one of the particles has to interact with another balance, for example by forming a transfer by creation from B to A (Fig. 2a). A pair is created with A having its asset and B having its liability, yielding a transfer of money. We cannot decide whether A created the currency and transferred the liability to B or B created the currency and transferred the asset to A. This is one of the reasons why we do not tie currencies to subjects but treat them as an independent entity. Currency units are abstract, their value is subjectively defined between trading subjects at the time they are used and have no direct connection to the traded good. Most restrictively, we could require that each new pair

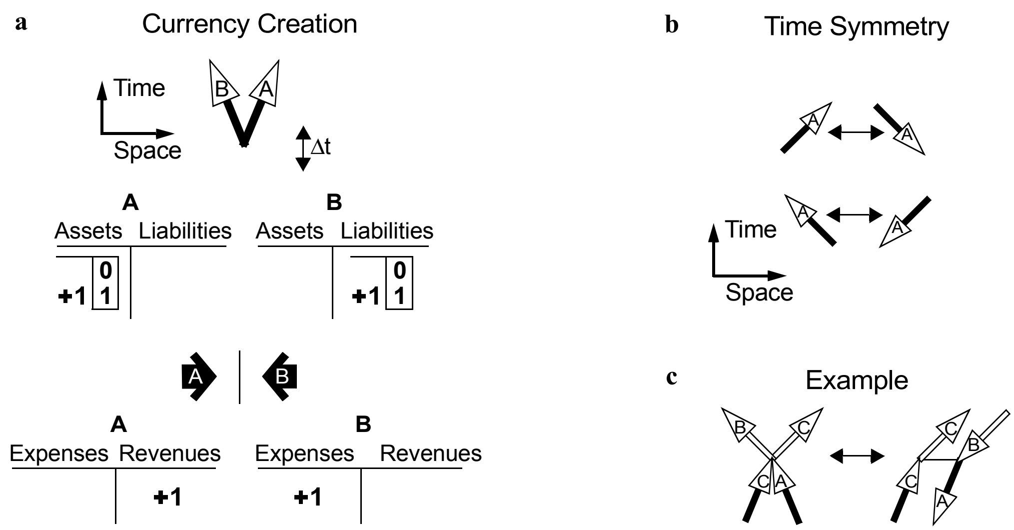

Fig. 2. ***Currency creation and time symmetry.*** (a) Currency is created by particle pair creation. It increases the energy but conserves the momentum. The black currency originates from the pair creation of A transferring to B by creation. The income statement is identical to an asset transfer from B to A. (b) Asset and liability particles show the same time symmetry as particles and antiparticles: an asset particle is indistinguishable from a liability particle going backwards in time. A liability particle cannot be distinguished from an asset particle going back in time. (c) The time symmetry allows for a more compact rewriting of space-time graphs where all particle paths have positive slope.

creation builds a novel currency which has to be exchanged against the existing ones. Less restrictive is the notion discussed in part 3, that the liability holding subject defines the currency which does not allow direct liability transfer. A least restrictive currency approach is a banking system which pools the currencies into two currencies in a bicurrency system as shown later.

Conclusion 5. Quantity and energy of money. Physically spoken, the pair creation of currencies (Fig. 2a) would not be possible since it increases the particle energy $E_{ic} = p_{ic}^2/2m$ with m the mass of the particle. Only in quantum mechanics this creation would have been possible for a short time Δt according to the energy-time uncertainty relation $4E_{ic}\Delta t \leq \hbar$. Interestingly, the definition of the quantity of money in economy is a linearized energy measure $M_{ic} = |p_{ic}|$. With this definition, we count all assets and all liabilities which means that we find a fourfold higher quantity than usually since we count beside bank liabilities also bank assets, non-bank assets and non-bank liabilities. Bookkeeping has no rule that conserves the quantity M_{ic}.

Furthermore, bookkeeping does not report increase or decrease of the quantity of money ΔM_{ic}. The income statement does not indicate it as seen by comparing Fig. 1d,e with Fig. 2a. In the following, we calculate from the momentum change Δp_{ic} the change in quantity ΔM_{ic}. We have to split the momentum change into four parts: revenue from increase of asset $\Delta p_{ic}^{a+} \geq 0$ or decrease of liability $\Delta p_{ic}^{l-} \geq 0$ and expense from decrease of asset $\Delta p_{ic}^{a-} \leq 0$ or increase of liability $\Delta p_{ic}^{l+} \leq 0$:

$$\Delta p_{ic} = \Delta p_{ic}^{a+} + \Delta p_{ic}^{l-} + \Delta p_{ic}^{a-} + \Delta p_{ic}^{l+} \quad (3)$$

The change of quantity ΔM_{ic} is then obtained by inverting the liability changes:

$$\Delta M_{ic} = \Delta p_{ic}^{a+} - \Delta p_{ic}^{l-} + \Delta p_{ic}^{a-} - \Delta p_{ic}^{l+} \quad (4)$$

This enables us to identify the subject i and the currency c which change the quantity, a fact probably important in discussions of inflation. Furthermore, the gross momentum transfer or transaction volume B_{ic} can be obtained from the absolute values of the changes:

$$B_{ic} = |\Delta p_{ic}| = |\Delta p_{ic}^{a+}| + |\Delta p_{ic}^{l-}| + |\Delta p_{ic}^{a-}| + |\Delta p_{ic}^{l+}| \geq 0 \quad (5)$$

From the gross transfer we derive both a transfer velocity v_{ic} and the realized exchange rate $x_{1,2}^{(i)}$ between currencies c_1 and c_2, which can be expressed as a transfer velocity ratio:

$$v_{ic} = B_{ic}/\Delta t \qquad x_{1,2}^{(i)} = \frac{B_{ic_1}}{B_{ic_2}} = \frac{v_{ic_1}}{v_{ic_2}} \quad (6)$$

Since the gross transfer is the sum of either asset transfers and liability transfers, v_{ic} can be split into an asset and a liability velocity. This extends traditional quantity theory which neglects the transfer of bank asset and non-bank liability. Traditional bookkeeping does not extract any of the variables ΔM_{ic}, B_{ic}, $x_{1,2}^{(i)}$ and v_{ic}. It thus discards relevant microeconomical information.

With above definitions, one can extract from each transaction the purchasing power MV of quantity theory on the left side of the quantity equation. However, there exists an intriguing unifying approach using the quadratic physical energy definition $E = p^2/2m$ with a particle mass m=1 for simplicity. Its time derivative $P = \Delta E/\Delta t$ is the physical definition of the power P. The absolute value of this power matches directly the MV term of the quantity equation for a transaction as seen from a short calculation:

$$|P_{ic}| = \left|\frac{\Delta E_{ic}}{\Delta t}\right| = \left|p_{ic}\frac{\Delta p_{ic}}{\Delta t}\right| = M_{ic}v_{ic} \quad (7)$$

This means that the quadratic energy definition $E_{ic} = p_{ic}^2/2$ unexpectedly makes sense in economics: the exchanged physical power in bookkeeping mechanics is the purchasing power of economics. Therefore, for a given transaction, we obtain MV of quantity theory for each currency c and subject i by calculating the energy E_{ic} before and after the transaction and dividing the absolute value of its change $|\Delta E_{ic}|$ by the transaction time. We thus see how the quantity of money and the transfer velocity can be merged into a single energy variable P_{ic}. Its accumulated ups and downs yield the left side of the quantity equation. This intriguing result bodes well for further analysis of quantity theory using bookkeeping mechanics.

Conclusion 6. Antiparticles and time inversion. If we compare the pair creation with physics, the asset particle would be the antiparticle of the liability particle and vice versa. In quantum electrodynamics, it is found that particles cannot be distinguished by their antiparticles going backwards in time (Richard Feynman (1985)). We can apply the same argument to bookkeeping mechanics. An asset particle with positive momentum is indistinguishable from a liability particle with negative momentum going backwards in time (Fig. 2b). This is due to the fact that the momentum sign changes under time inversion. Likewise a liability particle is an asset particle going backwards in time. This time symmetry allows for a more compact display of space-time graphs as shown in an exchange by recreation in Fig. 2c. To avoid confusion, we will not display graphs with time-inversed liabilities in this paper. The time symmetry shows that assets and liabilities would have equal characteristics if the time direction would have no meaning.

2. Exchange and Transfer Systematics

As a first application of bookkeeping mechanics, we give an overview over all possibilities to transfer and exchange between subjects using different currencies. This will serve as a basis for the subsequent analysis. We will describe the space-time graphs and bookkeeping of transfers between 2 subjects and 1 currency (4 cases), exchanges between 2 subjects and 2 currencies (10 cases), transfers between 3 subjects and 2 currencies (16 cases), exchanges between 3 subjects and 3 currencies (20 cases) and transfers between 4 subjects and 3 currencies (64 cases).

We have already treated two of the four basic ways to transfer between two subjects using a single currency: transfer of asset (Fig. 1c) and transfer by creation (Fig. 2a). In both cases, A obtains revenue and B has expense. Since an expense can either decrease asset (a-) or increase liability (l+) and a revenue can increase asset (a+) or decrease liability (l-), we find four possibilities as a result of a 2 x 2 combination matrix of expense versus revenue (Fig. 3a), numbered by roman numbers i, ii, iii and iv. The case (i) transfers asset, (ii) transfers by pair clearing, (iii) transfers by pair creation and (iv) transfers liability as shown in the space-time graphs and bookkeeping (Fig. 3a). Bookkeeping does not record the products or services which are bought by the transfers and are transferred in opposite direction. The force balance and the income statement is identical in all four cases: Subject A gains with a revenue force to the right and B looses with an expense force to the left (Fig. 3b). Nevertheless the quantity of money increases in the creation case iii and decreases in the clearing case ii. The choice, which of the four cases is chosen, depends on the initial momentum p_A and p_B as shown in Fig. 3c. These initial conditions are not equally distributed across all cases. For example, a pair clearing (ii) is performed if the liabilities of A and the assets of B are larger or equal the transferred units Δp, whereas the creation (iii) is performed when the assets and liabilities are equal or above zero. This

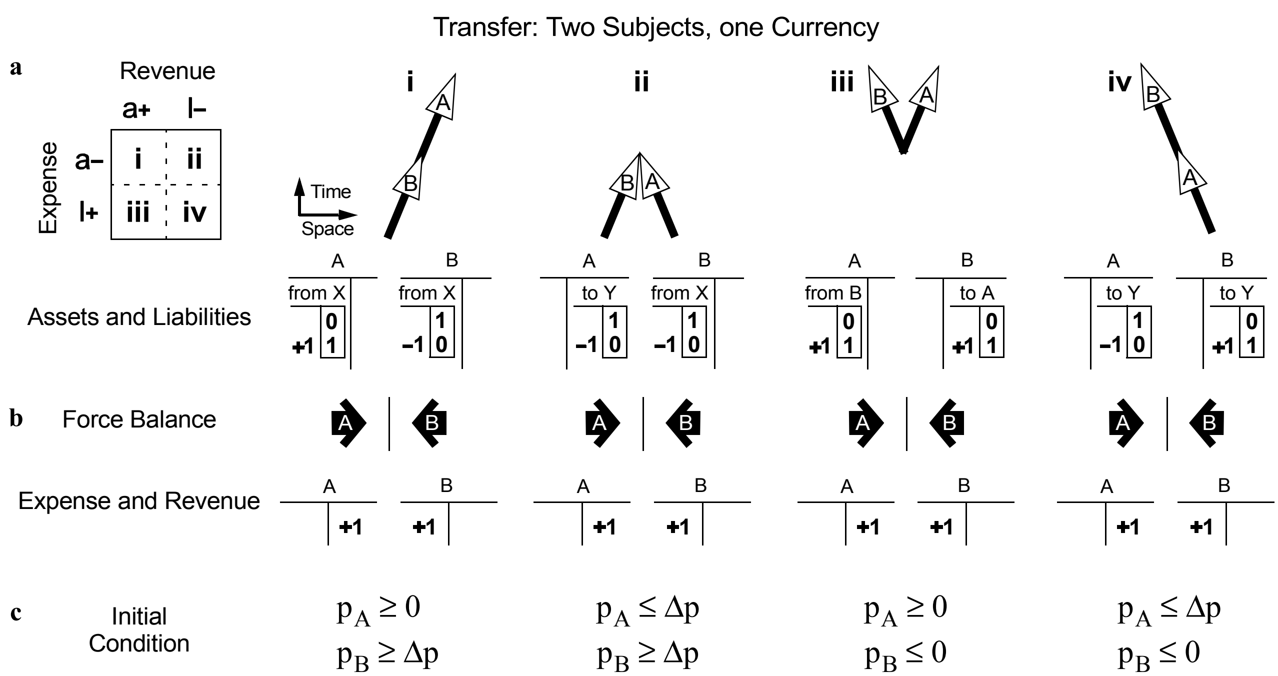

Fig. 3. ***Bookkeeping systematics: Transfer between two subjects with one currency.*** (a) The four combinations of expense with revenue by increase (+) or decrease (-) of asset (a) or liability (l): transfer of asset (i), transfer by clearing (ii), transfer by creation (iii) and transfer of liability (iv). (b) They yield an identical income statement. (c) The choice of the four cases depends on the initial conditions of momentum for A and B, which are not symmetrically distributed. The quantity of money i.e. the total energy is changed in the creation and clearing cases.

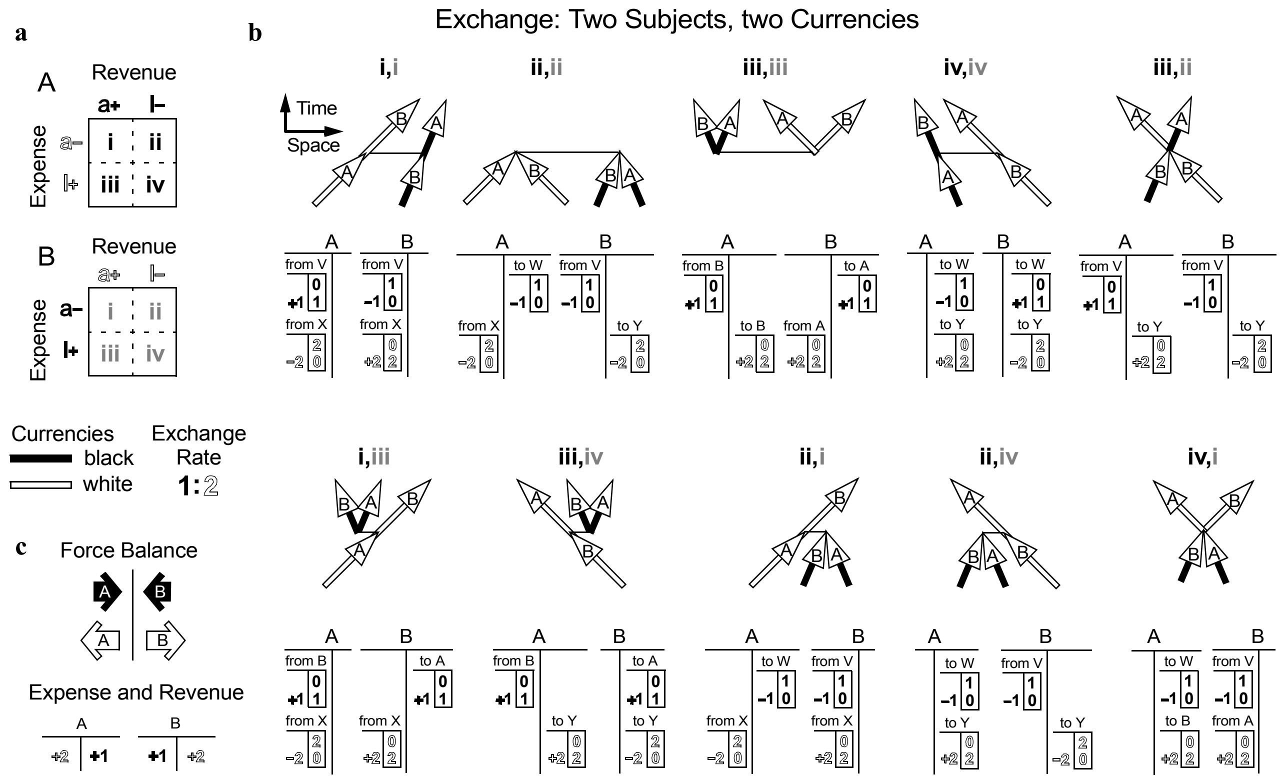

Fig. 4. ***Bookkeeping systematics: exchange between two subjects, two currencies.*** Exchanges between two subjects with two currencies show ten combinations: exchange of assets (i,i), of liabilities (iv,iv), of asset vs. liability (iii,ii), by double creation (iii,iii), by double clearing (ii,ii), by recreation (iv,i), by asset assisted creation (i,iii), by liability assisted creation (iii,iv), by asset assisted clearing (ii,i) and by liability assisted clearing (ii,iv). The case chosen depends on the initial stock of assets and liabilities of A and B. The quantity of money (i.e. the total energy) changes in the cases involving creations and clearings.

means that the change in the quantity of money M depends on an intricate interplay of transfer directions and initial conditions. Note that if the transferred amount is big enough to change the initial conditions, the transfer will be split into two or even three subtransfers.

We now consider the exchange possibilities between two subjects using two currencies. The prototype for this group is an exchange between assets (case i,i in Fig. 4b). There, two white currency units are transferred from A to B when one black currency unit is transferred from B to A. In the bookkeeping (Fig. 4b), the force balance and the income statement of it (Fig. 4c), we see two superimposed transfers of different currencies (see Fig. 3, case i). In this exchange, A and B came to terms that two white currency units at this time and this exchange situation had a value of one black currency unit. In the space-time graph we therefore see a flat white particle path and a steep black particle path. The momentum is conserved in each currency.

We find all combinations by entangling two expense-revenue matrices (Fig. 4a): A obtains revenue in black currency and expense in white, B obtains revenue in white and expense in black. This ensures identical force balances and income statements: black currency revenue for A and expense for B combined with white currency expense for A and revenue for B (Fig. 4c). An index pair of iii,ii means for example that we combined the iii-case for A and the ii-case for B. This results in an black asset transfer from B to A which exchanges with a white liability transfer from B to A. Since subjects can the exchanged in the asymmetric cases (ii,iii = iii,ii), the number of combinations is given by r=2 repeated and undistinguished drawings of k=4 numbers:

$$\tilde{C}_r^k = \binom{k+r-1}{r} = 10 \quad \text{(8)}$$

We name to the combinations. We find exchanges: of assets (i,i), by double clearing (ii,ii), by double creation (iii,iii), of liabilities (iv,iv), of asset versus liability (iii,ii), by recreation (iv,i), by asset assisted creation (i,iii), by liability assisted creation (iii,iv), by asset assisted clearing (ii,i) and by liability assisted clearing (iv,i). In each of them, the momentum is conserved in each currency (eq. 1), which means for example that the black arrow goes through the graph or that white pairs are created. As a consequence, the force balance adds to zero in each currency (Fig 4c). As in the case of Fig. 3, the initial availability of asset and liability determines the chosen case. Each of the cases have different impact on the total quantity M in each currency - some increase it, some decrease it, some keep it constant.

We give an overview over all combinations as space-time graphs in Fig. 5. We divide into transactions not transferring liabilities (left column) and transactions which transfer liabilities (right column). When going down in Fig. 5, the transactions become increasingly improbable since more and more subjects have to come to terms with their matching needs. The situation is similar to the treatment of an ideal gas in physics. Its properties can be solely derived by considering two particle collisions only, three particle collisions are so rare that they can be neglected in the calculations.

The transfers between three subjects and two currencies is given in Fig. 5c. It is constructed either by transferring an outgoing white assets of B to C or an incoming white liability from C to B (cases i',ii',iii',iv') or by either transferring an incoming white asset from C to A or one outgoing white liability from A to C (cases i",iii"). Since all combinations of the two expense-revenue matrices (Fig. 4a) are now distinguished, we find 4 x 4 = 16 transfer combinations. We name them and find transfers: by asset exchange (i,i'), by asset assisted transfer by creation (i,iii'), by asset transfer assisted creation (i,iii"), by asset transfer assisted clearing (ii,i'), by asset assisted transfer by clearing (ii,i"), by double clearing (ii,ii'), by double creation (iii,iii'), by asset transferring recreation (iv,i'), by liability transferring recreation (iv, i"). With allowing liability transfer we find transfer: by liability exchange (iv,iv'), by liability transfer assisted creation (iii,iv'), by liability assisted transfer by creation (iii,iv"), by exchanging asset versus liability (iii,ii'), by liability assisted transfer by clearing (ii,iv') and by liability transfer assisted clearing (ii,iv").

It should become clear from here on, how the complexity in transfers and exchanges develops further. For three subjects and three currencies, we combine three expense-revenue matrices and find $\tilde{C}_3^4 = 20$ different exchanges. We show four examples of them (Fig. 5d): threefold asset exchange (i,i,i), exchange by recreation versus asset transfer (ii,iii,i), double liability exchange versus creation (iv,iv,iii) and exchange by recreation versus liability transfer (ii,iii,iv). Going further in complexity, we expect 4 x 4 x 4 = 64 transfers between four subjects using three currencies, given with four examples (Fig. 5e): transfer by threefold asset exchange (i,i',i"), transfer by recreation versus asset transfer (ii,iii',i"), transfer by twofold liability exchange versus creation (iv,iv',iii") and transfer by recreation versus liability transfer (ii,iii',iv").

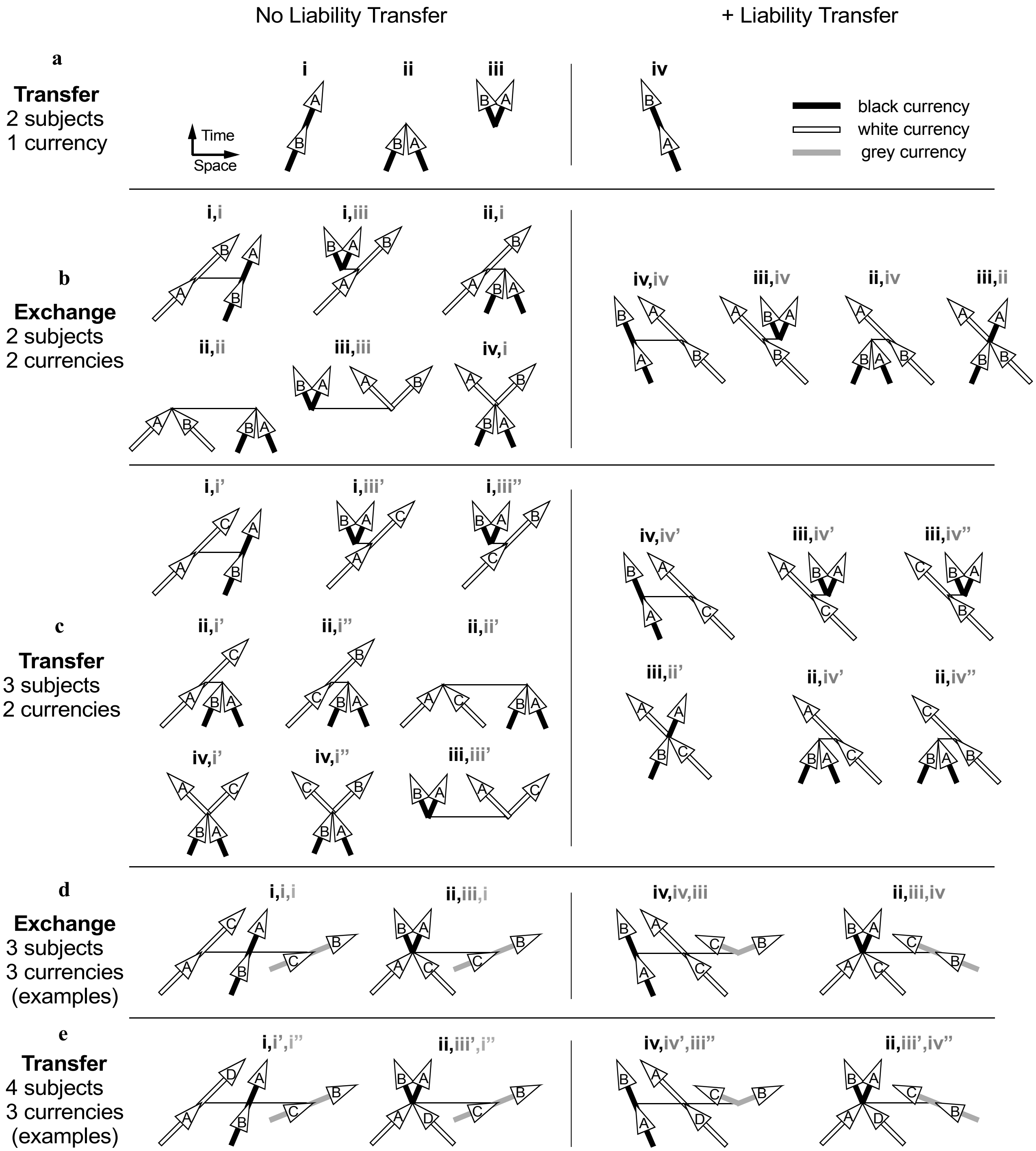

Fig. 5. ***Bookkeeping systematics.*** We show the transfers and exchanges of up to three subjects using up to three currencies and distinguish between combinations without liability transfer (left) and those using it (right). We find 3+1 transfers for two subjects and one currency, 10 exchanges for two subjects and two currencies, 16 transfers for three subjects and two currencies, 20 exchanges for three subjects and three currencies and 64 transfers for four subjects and three currencies.

Using the collected transfer and exchange systematics, we can decompose bookkeeping transactions into the shown basic building blocks. We do so in the following by analyzing the more commonly used transactions, namely the cash-free money transfer with and without a bank together with the bookkeeping of interest rates.

3. The hidden bicurrency system of banking

We approach bookkeeping of banks by discussing money transfer in historical and modern implementations of bookkeeping. Since money transfer in bookkeeping also incorporates the creation and reduction of loans, it will serve as the basis to analyze banking and the bookkeeping of interest rates.

Assume that three subjects A,B,C have their own currency white, grey and black (Fig. 6). We assume that the liability holder of a currency defines the value of a currency since he promises to make a back transfer in the future. Under these conditions, the subject can transfer value by asset transfer (Fig. 6 i), by transfer by clearing (Fig. 6 ii) and by transfer by creation (Fig. 6 iii). In the shown transfer, a currency unit of C (black) is very valuable, a unit of A (white) has medium value and a unit of B (grey) is considered least valuable. Under these conditions, N subjects create N currencies, yielding to N-1 exchange rates if the value of a currency is not further influenced by the asset holder from special relationships between the subjects, which would yield N^2-1 exchange rates. One can imagine that it is quite difficult to keep track of all these exchange rates.

Historically, a bookkeeping very similar to the above scenario is found in the usage of tally sticks (A. Mitchell Innes (1913), L. Randall Wray (1998), Georges Ifrah (1998)). For transfer by creation (Fig. 3, iii), two subjects notched currency units into a stick and split it into stock (asset) and stub (liability). The randomized fracture prevented one-sided counterfeiting very much comparable to modern split key encryption algorithms such as RSA or PGP. Splitting a tally stock and splitting a number into primes cannot be reconstructed with reasonable effort by another process. Other tally implementa-

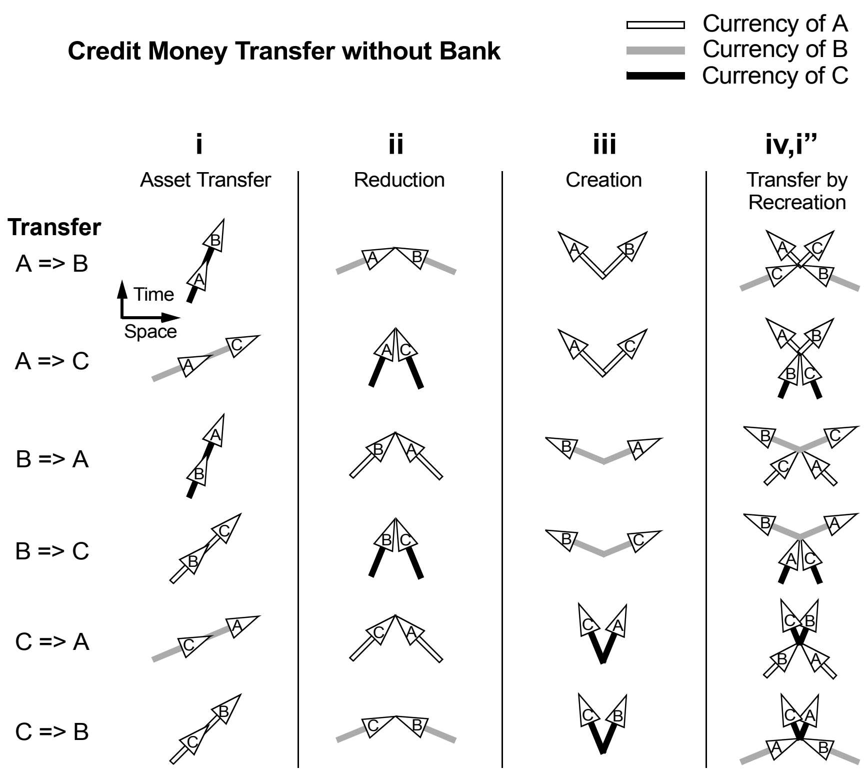

Fig. 6. ***Credit money transfer without a bank.*** We show a scenario of 3 subjects and three currencies where the value of a currency is defined by the liability holding subject. Transfers can occur either between two persons with an asset transfer (i), a transfer by reduction (ii) and a transfer by creation (iii). Otherwise, subjects have to transfer by recreation (iv,i”) since the asset holder has to change currencies as well. The low probability to perform the three person transfer reduces the liquidity of the liability with respect to asset.

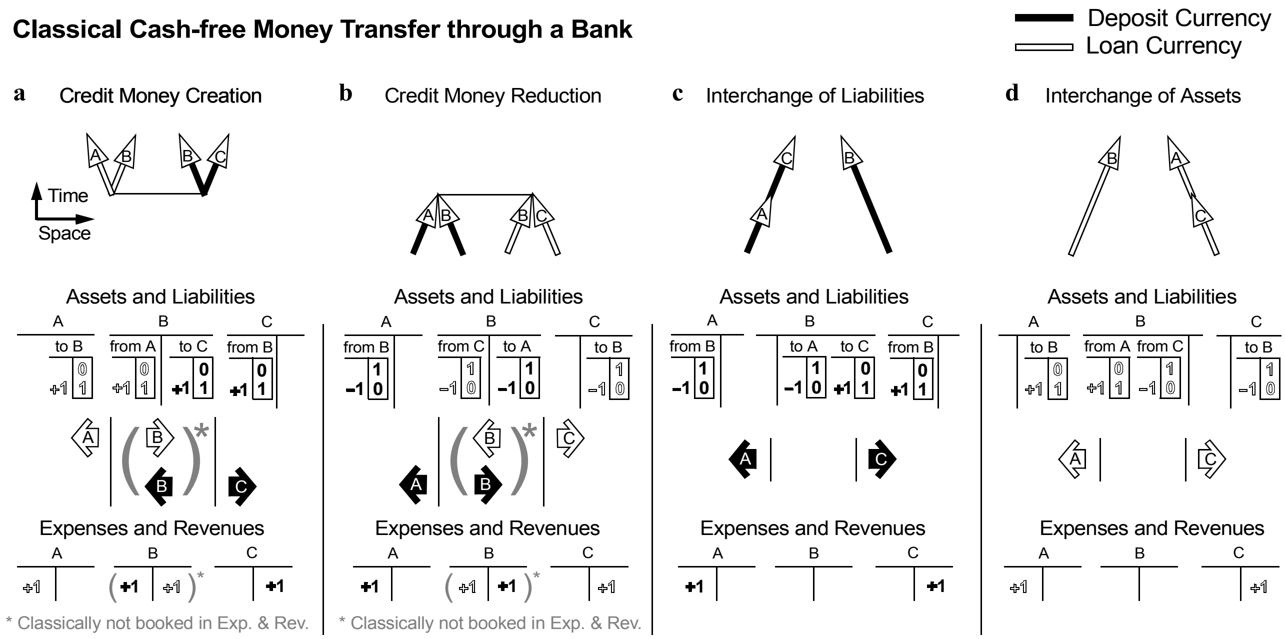

Fig. 7. ***Classical cash-free money transfer through a bank***. Subject A transfers to C through the bank B with four modes of cash-free money transfer: credit money creation, credit money reduction, interchange of liabilities and interchange of assets. Since we can divide each process into two processes between two subjects, we define two currencies: a black deposit currency for non-bank assets and bank liabilities and a white loan currency for bank assets and non-bank liabilities. In classical bookkeeping both exchange at a fixed exchange rate of 1:1 and are not distinguished.

tions were roman broken copper "cakes" (aes signatum) and babylonian broken clay sherds "contract tablets" (shubati tablets, the liability part sometimes found in fitting clay envelopes).

The transfer of liability (Fig. 3, iv) is not possible, since both stock and stub bear the name of the liability holder. A change of the liability owner would make the assessment of the currency value impossible for an asset holder. Note that we can nevertheless implement a transfer of liability, when the asset holder is changing synchronously to a newly created currency of the new liability holder, possibly with a different number of currency units (Fig. 6, iv,i"). This reduced liquidity of the liability would be another reason to switch to a centralized transfer institution of a banking system. A banking system condenses a system of N subjects and up to N currencies into a system of N subjects and 2 currencies, namely a deposit currency (black) and a loan currency (white) as shown in the following.

We analyze banking by focussing first on the four modes of cash-free money transfer from A to C via a bank B as documented in (Jürg Leimgruber (1992)). We show the space-time graphs, the bookkeeping, the force balance and the income statement (Fig. 7). Once again we find four modes of transfer: credit money creation, credit money reduction, interchange of liabilities and interchange of assets. Credit money creation is a double currency creation (see also Fig. 5c, case iii,iii'). Since two creations can in principle create two currencies, we distinguish between a white loan currency for non-bank liabilities and bank assets and a black deposit currency for bank liabilities and non-bank assets. This is the only possible choice, since otherwise the pair created by the bank would immediately clear again, deleting the bank B out of the transfer and yielding a simple transfer by creation (Fig. 5a, case iii). We furthermore confirm the two currencies by being able to split the transfer: the bank B is buying from C with a transfer by creation in the black deposit currency and sells to A with a transfer by creation in the white currency. In this case, we created in both transfers the same number of currency units. We will see in a minute, that this is only an arbitrary choice. The income statement for the bank reveals a white revenue and a black expense. They are not booked in the classical single currency bookkeeping,

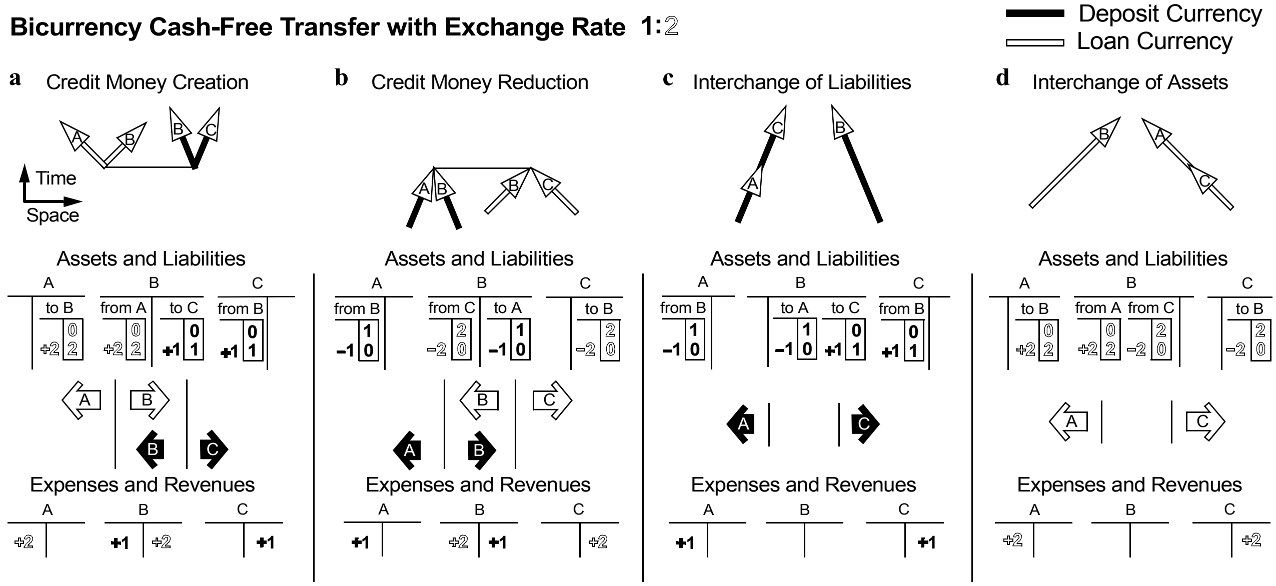

Fig. 8. ***Cash-free money transfer in a bicurrency system***. The two currencies in cash-free money transfer can have an arbitrary exchange rate. We show the graphs for the four modes now with an exchange rate of one black deposit currency units equalling two white loan currency units. In the creation and reduction cases, the banks and non-banks take an exchange rate risk of white versus black, seen by the force balance of the income statement.

since they cancel under the usage of a single currency. As for the transfer without a bank (Fig. 3), the choice of the case depends on the initial stock of asset or liability of A and C. The previous discussion whether M is increased or decreased by the transfer also applies here. We call the situation that the bank uses two currency groups the bicurrency system of banking.

The bicurrency system is also found in the other three cases. The second case of credit money reduction is applied when A has asset and C has liability. It can be split into B buying from C with a transfer by clearing in white and B selling to A with a transfer by clearing in black (Fig. 5c, case ii,ii'). The third case is interchange of liabilities where A transfers assets to C and the bank B merely shifts its liabilities in black from an account of A to an account of C. This spectating liability particle does not bounce another particle and could be also subtracted from the graph. Again the transfer can be split into a transfer from A to B and from B to C in black asset, not explicitly shown in the space-time, the force balance and the income statement since it cancels each other. The fourth case of interchange of asset, is completely symmetric to the interchange of liabilities in contrast to the discussion of Fig. 6. The bank negotiates and secures its assets obtained from loans to guarantee equal value in the transfer. Under this banking supervision and credit restriction, a transfer of non-bank liabilities has no effect on the value of the loan currency of the bank. The central institution of a bank therefore both condenses the multitude of currencies from N to 2 and enhances the liquidity of a non-bank liability transfer (Fig. 6, iv,i" vs. Fig 7d).

We can generalize cash-free money transfer to cases where the black and white currency do not exchange as 1:1 as discussed before and applied in the bookkeeping of today. We rewrite the four modes of cash-free money transfer in a situation where the exchange rate of two white currency units equal one black currency unit, i.e. black:white=1:2 (Fig. 8). Since all transfers can be subdivided into transfers from A to B and from B to C, we only find more flat white arrows, white "2" instead of white "1", doubled white forces and a doubled white revenue and expense. These changes do not conflict with the many-currency rules of bookkeeping mechanics. We have to imagine this transfer: A wants to buy something from C. Since both do not want to reveal their status as a deposit holder or a

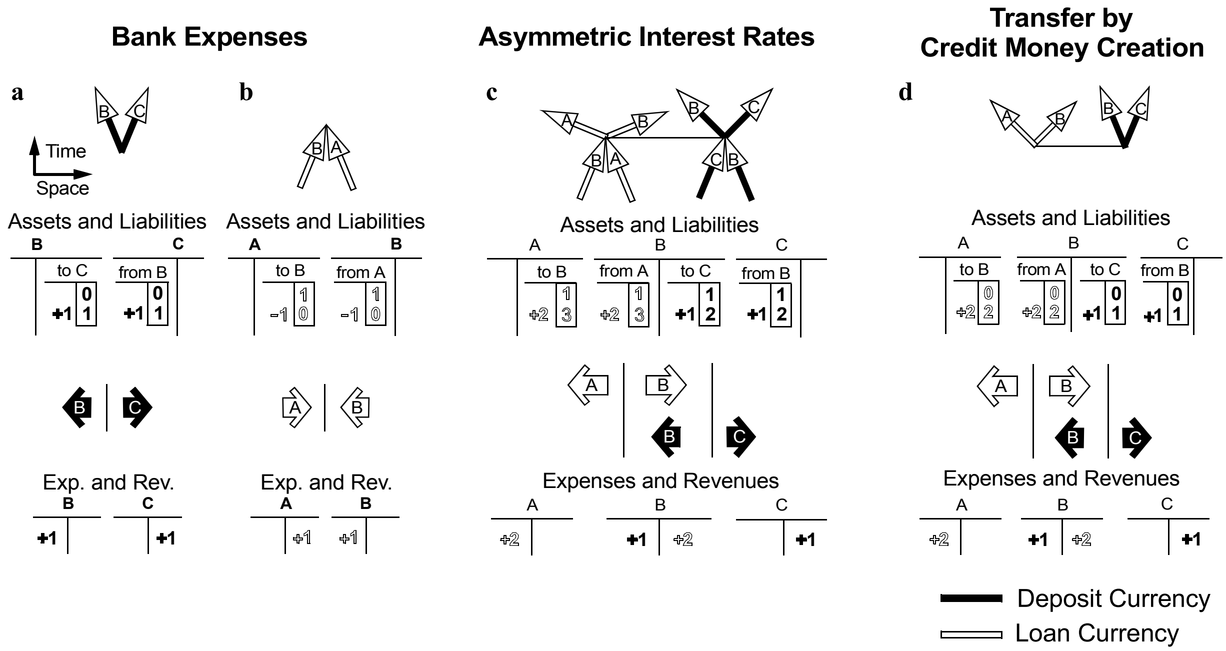

Fig. 9. ***Bank expenses and asymmetric interest rates.*** (a,b) Banks pay expenses either with a transfer by creation in the black deposit currency or with a transfer by clearing in the white loan currency. (c) Both can be balanced with a booking of asymmetric interest rates under the assumption of a constant exchange rate between black and white. (d) The booking of asymmetric interest rate is identical to a transfer by creation with an exchange rate between black and white given by the interest rates. In this case, the bank can issue no profit from the booking of the asymmetric interest rate.

loan holder, they will settle for two prices: a black price in the deposit currency and a white price in the loan currency. The banking system and a whole bunch of economic variables will probably affect the exchange rate between both prices. In the transfer, both A and C will not be able to distinguish, whether their own change in deposit or loan came from a change in deposit or loan of the transfer partner. Likewise the bank will not tell, which of the four modes it applied securing the secrecy of banking. On the other hand, this means that banks internally apply a two-price market, which is fixed and not accessible by the markets from outside.

We discuss how this exchange rate influences the profit of the bank. A bank needs revenues to provide the service of a central liability insurance and a clearing house. When the bank has expenses toward a deposit holder C, it books a transfer by creation which increases only the quantity of black deposit currency (Fig. 9a). If the bank pays a loan holder A, it does so with a transfer by clearing, decreasing only the quantity of the white loan currency (Fig. 9b).

The bookkeeping which typically balances these expenses, is the booking of asymmetric interest rates (Fig. 9c) as transcribed from (Jürg Leimgruber (1992)). We assume in the example an exaggerated 100%/Δt interest rate for deposits and 200%/Δt for loans. This translates to a 2-fold increase of the black currency quantity and a 3-fold increase in the white currency quantity. It can be interpreted as a transfer by creation: two white currency units are transferred from A to B and one black currency unit is transferred from B to C. We thus see that the booking of asymmetric interest rates is equivalent to the cash-free money transfer case of credit money creation with an exchange rate of black:white=1:2 (Fig 9d, Fig. 8d). Note for example the identical force balances between Fig. 9c and Fig. 9d. Yet in the transfer case (Fig. 9d), nobody would have assumed that the bank had profited, since two white revenue units of the bank are supposed to have the same value as one black expense unit.

We learn two things out of this. First, an asymmetric interest rate provides for the bank only revenue, if the exchange rate between the black deposit and the white loan currency is fixed to 1:1 or at least below the asymmetry of the interest rate. It is tempting to make the following argument: the asym-

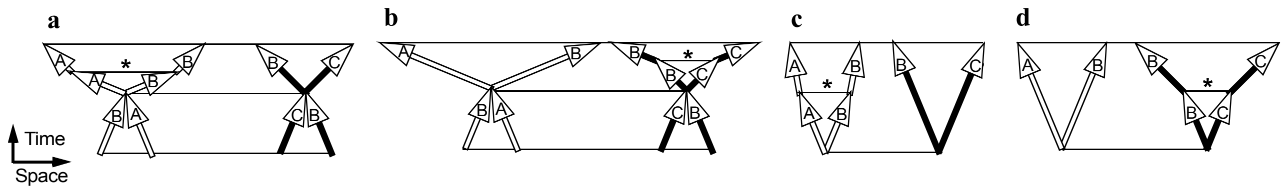

Fig. 10. ***Balancing bank expenses.*** We show four scenarios of how to balance bank expenses. (a,b) Bank expenses to either A in white or C in black can be balanced with asymmetric interest rates if the final exchange rate when C is paying back to A does not change. (c,d) Bank expense can be balanced directly without asymmetric interest rates by a change in exchange rate from an initial 1:1 to a final 1:2 between white and black.

metric increase in quantity of money (2 to 4 in black, 2 to 6 in white) from asymmetric interest rates would, following quantity theory at constant velocities for black and white, move the market to the exact exchange rate of 2:3 at which C can pay back to A without having to provide additional assets. This means the bank cannot obtain a profit. And second, by turning around the argument, we see that in an established bicurrency system, a bank can profit from a decreasing exchange rate of white:black without having to raise asymmetric interest rates at all.

We show four fictive example scenarios for both ways of bank profits in Fig. 10a-d. All contain the following transactions: A transfers to C by creation with at an exchange rate of 1:1, the bank has expenses and buys either from A in white (* in Fig. 10a,c) or from C in black (* in Fig. 10b,d) and C finally transfers back to A. In the first two scenarios (Fig. 10a,b), we apply asymmetric interest rates and the bank pays its expenses since B transfers back to C with an exchange rate of 1:1. This makes sense, since the quantity of money after the bank payments to A or C is still white:black=4:4 (Fig. 10a) or 6:6 (Fig. 10b) - yet the application of quantity theory is hampered by the fact that the transfer velocity was higher in white (Fig. 10a) or in black (Fig. 10b). Note that in both cases, price levels have to adapt to the increased money quantity in the final back transfer from C to A. Yet interestingly, the quantity is increased differently depending on from whom the bank buys products (from 2 to 4 in Fig. 10a or from 2 to 6 in Fig. 10b).

In the second two cases (Fig. 10c,d), we balance the bank expenses by changing exchange rates: the expenses of the bank B towards A (Fig. 10c) or towards C (Fig. 10d) are possible, because the exchange rate drifted from 1:1 to white:black=1:2 later when C transfers back to A. This drift gives the bank B the possibility to profit from a decreasing exchange rate in a most basic bicurrency scenario. Again we can only partly explain the drift in exchange rate from quantity theory: the bank payments adapt the quantities for the final back transfer. Yet again we would ignore issues of transfer velocity in doing so.

To conclude, we have revealed a bicurrency system in banking as we translated its bookkeeping to a multi-currency analysis of bookkeeping mechanics. Two currency groups - deposit and loan currency - can have arbitrary exchange rates, possibly changing the impact of interest rates. At this point, we could only give an analysis from the view of bookkeeping logics, an economical model analysis yet has to be undertaken in the light of these findings. The analysis is an example of how economic discussions can connect to bookkeeping with the graphs of bookkeeping mechanics.

4. Leaky Bookkeeping: violations of the zero-sum game

All our bookkeeping analysis so far implemented the realization principle which states that all prices are realized between subjects. This implies that assets given by a subject are received by another subject. As discussed in part 1, this then implies conservation of momentum, a zero force balance across subjects and a zero-sum game for each currency. Since a bookkeeping thus only interacts with other bookkeepings, the world is divided into the world of debt information in bookkeeping and the material world of products and services. Subjects negotiate and trade products and services from within a leak-proof 'sandbox' of interconnected bookkeepings. This means also that the material world or 'nature' cannot interact directly with the bookkeeping, only indirectly via subjects and their ability to negotiate.

Surprisingly, we find bookkeeping transactions, where the above picture does not hold at all. We are not speaking of counterfeiting or loss of bookkeeping information. We will show how statements of capital, the bookkeeping of depreciations and single-sided exchanges directly violate the realization principle. They create assets or liabilities which are not balanced by other assets or liabilities. They

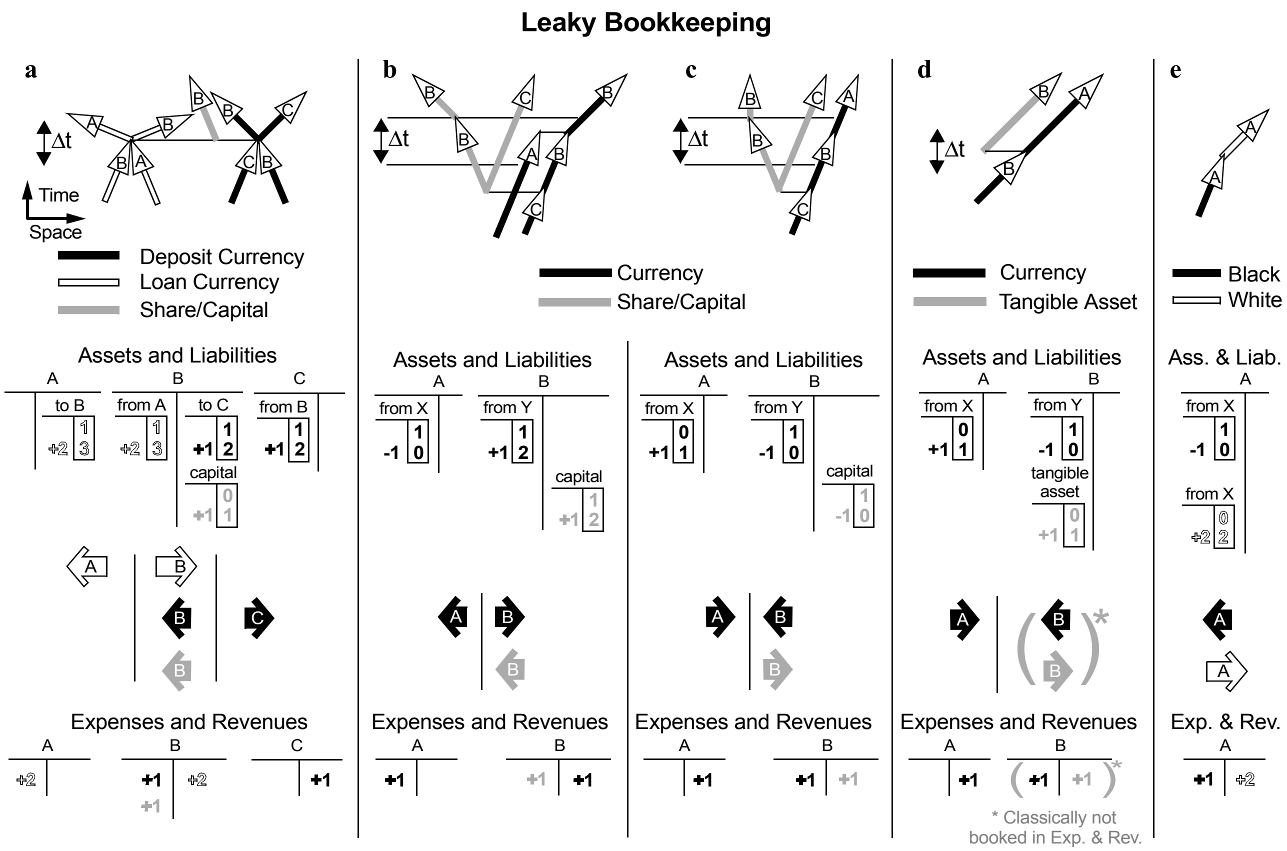

Fig. 11. ***Statement of capital, depreciation and single-sided exchange violate the zero-sum game***. (a) A statement of capital after interest rate application. The bank artificially balances its revenue between black and white with a grey capital liability. Violating the zero-sum game, this unrealized compensation reduces the overall momentum in the grey capital liability. (b) A statement of capital of an incorporated company B, created by C. B has gained from A within the fiscal time span Δt. The 'balancing' of revenue in capital liability violates the zero-sum game. (c) When the company B has lost, overall momentum is now increased in the grey share/capital currency. (d) The beginning of a depreciation of a freshly produced good, bought by B from A increases the total momentum and violates the zero-sum game in the grey depreciation currency. (e) When a currency exchange is only applied in a single bookkeeping it is not realized against another subject. The result is a zero-sum game violation in both currencies.

therefore fail the zero-sum game and its logic equivalences of the conservation of momentum and a zero force balance.

Statement of capitals are a consequence of the believe that bookkeepings must be balanced. This artificial symmetry is achieved by a statement of capital which adjusts the account of capital such that assets equal liabilities. But as we discussed and saw, the bookkeeping was already counterbalanced by all the other bookkeepings by virtue of the zero-sum game and the realization principle. Both principles are violated by a statement of capital as shown below.

We begin with the example of the statement of capital of a bank after application of asymmetric interest rates (Fig. 11a) as documented in (Jürg Leimgruber (1992)). The statement balances the profit of the bank bookkeeping by introducing an increase in capital liability in the grey capital/share currency, not counterbalanced by another bookkeeping since the bank bookkeeping was already counterbalanced by the bookkeepings of A and C before. This is a failure of the zero-sum game, easily seen in the non-zero force balance in the grey currency. Note that many non-banks do not perform a statement of capital which could re-establish the zero-sum game. (Even then the quantity of the grey currency would increase, depending on the absolute sum of profits and losses within a financial time span which would not have an economical motivation.) The number of assets in the grey share/capital currency are not increased by virtue of the statement of capital. Even if they would be increased to save the zero-sum game, the following circularity arises: an increase in share assets would at the same time change another balances statement of capital. Through a series of bookkeepings this iteration has the potential to change the bank's statement of capital, since it might hold share asset units of another company in a circular fashion. It is now unclear how this circularity should be updated over time: parallel or by tracking N dependencies or by hoping that an infinite tracking would find a series of converging statements of capital. Each would yield a different output.

The same failure of the zero-sum game is demonstrated for two statements of capital of an incorporated company B. In both cases the company was established before by C with an investment using a black currency asset, yielding a pair creation of share assets and company capital in grey. The company B gains (Fig. 11b) or looses (Fig. 11c) asset in black currency from A in a given fiscal time span Δt. We assume again that B issues a statement of capital whereas A does not. Again, the change of capital liability of B from the statement of capital negatively doubles revenues (Fig. 11b) or expenses (Fig. 11c), which is not realized against any other subject.

In all three statements of capital, we see how the assumption that a single bookkeeping has to be balanced leads to a single-sided increase of the capital account. Artificially, the revenue or expense force over a fiscal time span changes the momentum of both the other subject's bookkeepings and the momentum of the capital account. This doubling of momentum change is not governed by bookkeeping mechanics and has the same result in the bookkeeping as if the increase or decrease in capital liability was counterfeited.

The violation of the realization principle is quite similar in depreciations. Assume subject B has bought a tangible asset from A with one black currency unit. B now accounts for the tangible asset in his depreciation account using one currency unit marked as grey (Fig. 11d). We make the realistic assumption that the tangible asset was newly produced and therefore not yet accounted for in a depreciation account of A. Thus the asset in the grey currency is not counterbalanced by an asset before the purchase. The new asset in the hatched tangible asset currency is not realized against another subject. The momentum of B has decreased without increasing the momentum of A accordingly. We thus again detect a failure of the realization principle, leading to a violation of the zero-sum game. Again it was motivated by 'balancing' a single bookkeeping. In the income statement this results in a revenue force in grey currency to the right, not balanced from other bookkeepings. In the depreciation case,

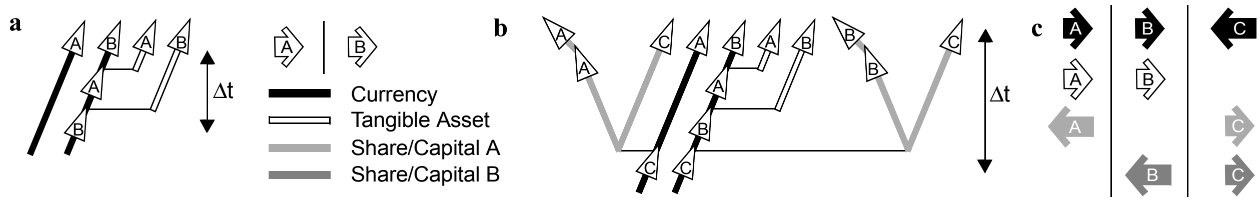

Fig. 12. ***Violations of the zero-sum game copies depreciations into the capital account.*** (a) Two subjects A and B sell a tangible asset to each other and both yield a profit by a violation of the zero-sum game. (b) The same situation implanted into two corporate companies founded by C, results in an increase of the capital account by a statement of capital. (c) The income statement shows how both violations balance each other in different currencies.

the law does not allow everybody to depreciate and increase its asset in this way. A private person for example cannot depreciate a new car.

The inverse situation occurs after the initial depreciation when depreciations reduce the tangible asset in the depreciation account with a legally prescribed depreciation rate. Again the depreciation is not realized against another subject and violates the zero-sum game and the conservation of momentum, this time by reducing the total momentum. Even for long time scales, the zero-sum game is not necessarily regained since the positive momentum violations from new depreciations do not necessarily balance the negative violations due to the decaying depreciation.

We want to sketch how depreciations act on a statement of capital. Consider two subjects A and B each selling to one another a tangible asset. This results in a doubled increase of the depreciation account making the violation of the zero-sum game very obvious. Both A and B make a profit without anyone yielding a loss (Fig. 12a). If this depreciation is implanted into the situation of two corporate companies A and B, both founded by C, we see that the profit from the depreciations is copied into the capital account with the statement of capital (Fig. 12b). Since the statement of capital imbalances the share/capital currency, it appears as if the violations of the zero-sum game of depreciations and statement of capital balance each other in different currencies as seen in the force balance income statement of both processes (Fig. 12c). For example the increase in tangible asset in white is ’balanced’ by an increase in the capital account in grey. Note that the imbalance in the share/capital currency will at some point be used to issue new shares by the companies. We thus see how depreciations increase the capital account of two companies without decreasing the capital of other companies. The competition between companies is not any more capped by the zero-sum game, but by the amount and speed of fresh depreciations which are not balanced by ongoing depreciations.

We would argue that depreciations are remnants of a bookkeeping which yielded an inventory of goods measured in physical units (e.g. kg, liter, hours) to measure unrealized ’value’ in advance of its monetary realization (see for example Luca Pacioli (1494)). We would therefore suggest to split bookkeeping into a monetary bookkeeping under the rule of the realization principle and a company database for tangible assets to form a resource planing (flow) and an inventory (stock).

We find yet another way to violate the zero-sum game. When currencies are exchanged in a single bookkeeping, they are often not realized against another subject. As an example, subject A changes an asset of one black currency unit into two white currency units (Fig. 11e). Since classical bookkeeping does not allow to account for many currencies, foreign currencies are noted in the bookkeeping by virtue of above graph using a non-realized exchange rate. This exchange is not realized against an exchange partner with a reverse transfer to give the exchange case i,i of Fig. 5b. Therefore a momentum imbalance in both the white and the black currency is introduced. In the example, the black currency loses momentum and the momentum of white is increased. This creates a violation of the zero-sum game in each currency by violating the realization principle.

We have shown peculiar examples of bookkeeping which violate the rules of the zero-sum game. Since nearly all economic modelling presupposes a zero-sum game, an economic discussion of above leaky bookkeeping will prove interesting and illuminating.

4. Conclusions

We have developed a new physical approach to analyze bookkeeping. One of the motivations in the beginning was find a clear graphical description of bookkeeping to clarify and enhance economic discussions. One has to understand that the space-time description of Feynman-graphs was groundbreaking in physics to understand and calculate the interactions between photons and electrons. Although Feynman-graphs were first devised for quantum electrodynamics, they are now applied to the elementary particles of the standard model and are widely used to calculate processes in semiconductor and solid state physics. Besides providing a complete space-time picture of a process, Feynman-graphs describe axiomatic building blocks which allow for recursions, iterations and decompositions between given inputs from the past and expected outputs in the future. We describes how the powerful space-time Feynman-graph approach can be applied to bookkeeping. Bookkeeping is thus decomposed into basic transfer and exchange graphs of currencies between subjects (Fig. 5). All transactions can be derived from them.

Besides being able to directly translate between physics and bookkeeping and thus connecting physics more tightly with economics and economics more tightly with bookkeeping, we discussed two new findings obtained from the interdisciplinary approach of bookkeeping mechanics. First, we identified two currency groups in banking (part 3) and second we argued towards a multi-currency realization principle (part 4). More is to be expected from the translation scheme of bookkeeping mechanics in the future.

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